

Analogy or Induction? Translating and interpreting Li Shanlan's Argumentative Mode

Andrea Bréard

Université Paris-Sud
Faculté des Sciences d'Orsay

Karl Jaspers Center for Advanced Transcultural Studies,
Ruprecht-Karls-Universität Heidelberg

October 10, 2018

Zhang Yitang 張益唐, *Annals of Mathematics* (2014)

It is proved that

$$\liminf_{n \rightarrow \infty} (p_{n+1} - p_n) < 7 \cdot 10^7,$$

where p_n is the n -th prime.

Zhang Yitang 張益唐 (1955 –)



Five “Exercises in Style”

- I The Success Story
- II Presentism
- III Global History
- IV Nationalist
- V Unrigorous



The Success Story

Li Shanlan 李善蘭 (1810-1882)





Presentism

The “Li Shanlan Identity”

$$\sum_{j=0}^k \binom{k}{j}^2 \binom{n+2k-j}{2k} = \binom{n+k}{k}^2$$



Global History

科學專著

垛積比類疏證

章 用

Zhang Yong, Proofing the formulas in the *Analogically arranged Accumulations of Piles*, *Kexue* 23 (1939)

MATEMATICKO-FYZIKÁLNY ČASOPIS SAV, 12. 3. 1962

POZNÁMKA K JEDNOMU ČLÁNKU P. TURÁNA

JOSEF KAUCKÝ, Bratislava

1. P. Turán v článku [1] dokazuje kombinatorickou identitu

$$(1) \quad \sum_{j=0}^k \binom{k}{j}^2 \binom{n+2k-j}{2k} = \binom{n+k}{k}^2,$$

kterou v roce 1867 bez důkazu uveřejnil čínsky matematik Le-Jen Shoo.

Josef Kaucký, Remarks on a work by Turán, *Matematicko-Fyzikálny Časopis* (1962)

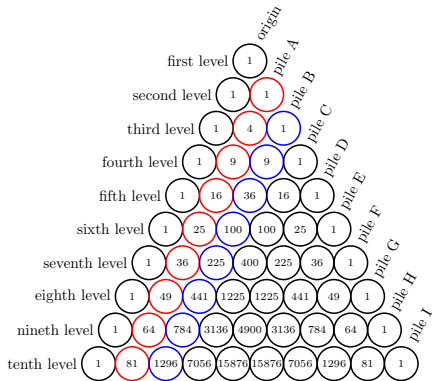
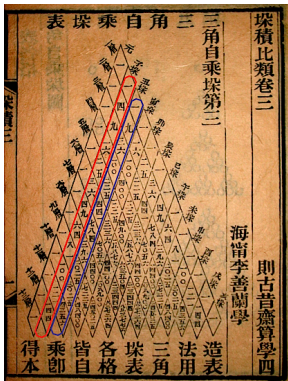
IV

Nationalist

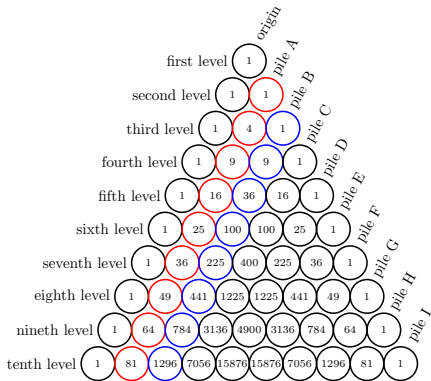
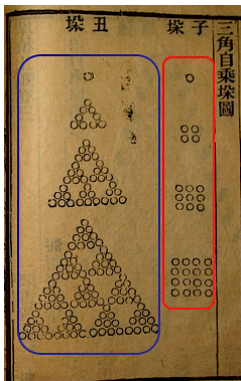
V

Unrigorous

Li Shanlan, *Duoji bilei* 垛積比類 (1867)



Li Shanlan, *Duoji bilei* 垛積比類 (1867)



Li Shanlan, *Duoji bilei* 垛積比類 (1867)

三角自乘垛者三角垛逐層皆自乘也子垛爲一乘垛逐層自乘之其積丑垛爲二乘垛逐層自乘之其積寅垛爲三乘垛逐層自乘之其積卯垛以下可類推

三角自乘垛有層求積術

子垛有方一隅一方以層爲高隅以層減一爲高各以三角二乘垛求積術入之

丑垛有方一廉四隅一方以層爲高廉以層減一爲高隅以層減二爲高各以三角四乘垛求積術入之

寅垛有方一甲廉九乙廉九隅一方以層爲高甲廉以層減一爲高乙廉以層減二爲高隅以層減三爲高各以三

Li Shanlan, *Duoji bilei* 垛積比類 (1867)

$$\sum_{k=1}^n \binom{k}{1}^2 = 1 \cdot \binom{n+2}{3} + 1 \cdot \binom{n+1}{3}$$

$$\sum_{k=2}^{n+1} \binom{k}{2}^2 = 1 \cdot \binom{n+4}{5} + 4 \cdot \binom{n+3}{5} + 1 \cdot \binom{n+2}{5}$$

$$\sum_{k=3}^{n+2} \binom{k}{3}^2 = 1 \cdot \binom{n+6}{7} + 9 \cdot \binom{n+5}{7} + 9 \cdot \binom{n+4}{7} + 1 \cdot \binom{n+3}{7}$$

$$\sum_{k=4}^{n+3} \binom{k}{4}^2 = 1 \cdot \binom{n+8}{9} + 16 \cdot \binom{n+7}{9} + 36 \cdot \binom{n+6}{9} + 16 \cdot \binom{n+5}{9} + 1 \cdot \binom{n+4}{9}$$

$$\sum_{k=j}^{n+j-1} \binom{k}{j}^2 = ?$$

procedure to find the sum for a given n 有高(or: 有層(數))求積術

reference	Type of 'accumulation'	# cases explicitly given for specific n	procedure for general n
<i>juan</i> 1:1B-5B	三角垛	4 (一乘垛... 四乘垛)	procedure explicitly stated
<i>juan</i> 1:6A-10A	一乘支垛	5 (方垛 + 4)	第五垛以上可類推
<i>juan</i> 1:10B-15B	二乘支垛	5 (方垛, 甲垛 + 3)	第四垛以下可類推
<i>juan</i> 1:15B-20A	三乘支垛	6 (方垛, 甲垛, 乙垛 + 3)	第四垛以下可類推
<i>juan</i> 1:13A	三乘支垛 以下理俱同	-	-
<i>juan</i> 2:1A-7B	乘方垛	6 (太垛, 元垛 + 4)	五乘方垛以上遞增一廉各廉之數詳左表餘法可類推
<i>juan</i> 2:7B-14B	二乘方支垛	7 (方垛, 甲垛 + 5)	第六垛以下可類推
<i>juan</i> 2:15A-26A	三乘方支垛	9 (方垛, 甲垛, 乙垛 + 6)	第七垛以下可類推
<i>juan</i> 3:1A-6A	三角自乘垛	4 (子垛, 丑垛, 寅垛, 卯垛)	辰垛以下可類推
<i>juan</i> 3:6B	子支垛		
<i>juan</i> 3:6B	丑支垛		
<i>juan</i> 3:7A-19B	寅支垛	8 (方垛, 甲垛, 乙垛, 丙垛, 丁垛 + 3)	第四垛以下可類推
<i>juan</i> 3:20A-36B	卯支垛	9 (方垛, 甲垛, 乙垛, 丙垛, 丁垛, 戊垛, 己垛, 第一, 二垛)	第三垛以下可類推
<i>juan</i> 4:1A-7B	三角變垛	5 (第一垛... 第五垛)	第六垛以下可類推
<i>juan</i> 4:8A-15B	三角再變垛	5 (第一垛... 第五垛)	第六垛以下可類推
<i>juan</i> 4:16A-23B	三角三變垛	5 (第一垛... 第五垛)	第六垛以下可類推

Li Shanlan's text contains

- ▶ a deductive structure. It logically builds up results that he refers back to permanently, and, not unlike Euclid's *Elements*, it is self-contained - **YES**
- ▶ proofs that convince but do not explain - **NO**
- ▶ proofs that in addition to providing the required conviction that the result is true also show why it is true - **YES and NO**
- ▶ invitations to think the problem through to the general case - **YES**.

The latter would certainly have pleased George Pólya (1887–1985), it suits Pólya's ideas about teaching the art of plausible reasoning in mathematics: good guesses that precede rigorous mathematical proofs. But this was not the case in the eyes of late Qing Chinese mathematicians. Their standards of mathematical validity had a different format: modes of inductive argumentation were both visual and rhetorical, and the authority of these modes of argumentation was built upon both Chinese philosophical and foreign elements.