Mathematics as/in science

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Book of abstracts

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About the workshop

he relationship between mathematics and science continues to be of considerable philosophical interest. Within contemporary philosophy of science, for example, pinpointing the exact role of mathematics in the sciences remains a hotly debated issue. Does mathematics play a mere inferential role in that it allows for the derivations of one substantial truth from another or is mathematics more than a theoretical juice-extractor? Are there distinctive mathematical explanations of physical phenomena? Similar questions can be asked about the role of logic in science.

These issues connect with discussions within the philosophy of mathematics (and the philosophy of logic) concerning the nature of mathematics (or logic). Within the philosophy of mathematics, Platonists, nominalists and structuralists consider mathematics to be fundamentally different in kind from empirical science, while empiricists have argued that mathematics is, just like other sciences, fundamentally about aspects of the empirical world. Different positions within the debate about the nature of mathematics will, arguably, lead to different answers to the question as to how mathematics and science are related.

In this workshop we want to focus on how these different philosophies of mathematics fare in giving an account of mathematical practice and the role of mathematics in scientific practice.

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Abstracts

Invited speakers

Victor Gijsbers (Leiden University)

Causal Interventionism for Science, Grounding Interventionism for Mathematics?

In a 2017 paper, I proposed a theory of mathematical explanation inspired by Woodward's interventionist theory of causal explanation in the sciences. According to this theory, explanatory asymmetries in mathematics result from 'quasi-interventions' on mathematical objects that have to be understood in terms of interventions on our mathematical practices. This theory makes mathematical explanation subjective in the sense that explanatory asymmetry depends on contingent features of our practices.

In the original paper, I briefly discuss the possibility of using an objective relation of grounding to generate objective asymmetries, only to dismiss this. The current paper reconsiders this quick dismissal in the light of work by Jonathan Schaffer and Alistair Wilson on the relation between grounding on the one hand and interventionist theories of causation on the other hand. Can grounding interventionism – as Amanda Bryant has recently called the emerging position – be to mathematical explanation what causal interventionism is to scientific explanation? I argue, first, that such a theory has some hope of success in the sense that it can at least deal with Marc Lange's counterexamples to the use of grounding explanation in mathematics. But I also argue, second, that Bryant's epistemological worries about grounding interventionism (which have no direct counterpart in the sciences) lead us right back to the subjectivity which we may have hoped to avoid.

Joachim Frans and Bart Van Kerkhove (Vrije Universiteit Brussel)

Exploring disagreements about explanatoriness in mathematics.

Mathematicians suggest that some proofs are valued for their explanatory value. This has led to a philosophical debate about the distinction between explanatory and non-explanatory proofs. In this talk, we explore whether disagreements about the explanatory value of a proof are possible and how to understand these disagreements. By considering an epistemic and contextual conception of explanation, we suggest that we can make sense of disagreements about explanatoriness in mathematics by identifying differences in the epistemic aims of mathematicians or mathematical communities. We illustrate this relation between explanation, epistemic aims and disagreements by looking at cases from mathematical practices.

Valeria Giardino (Institut Nicod)

Experimenting with Triangles.

There has been a long and extensive debate in philosophy about the functioning of thought-experiments in science (see Stuart et al. 2018). However, less work has been devoted to the specific case of thought-experiments in mathematics. Are there genuine thought-experiments in mathematics? According to Lakatos for example, proofs can be seen as thought-experiments suggesting a decomposition of the original conjecture into lemmas (Lakatos 1976). Would that mean that all reasoning in mathematics can be presented as a thought-experiment? In my talk, I will analyze the notion of thought-experiment in mathematics by considering three examples of reasoning with triangles (Klein, 1903; Giaquinto 2007; Bråting and Pejlare, 2008). This will bring me to the definition of a framework where mathematics is considered as threefold, in a continuous interaction between theory, experiment and technology (Rav, 2005); only some (thought) experiments can be considered as proofs, thanks to the constraints built-in or notified on the representations involved.

Contributed Talks

Viktor Blasjo (Utrecht University)

The operationalist philosophy of ancient Greek geometry

Ancient Greek geometry was heavily focussed on constructions. Throughout the centuries when Greek mathematics was at its peak, the best mathematicians proposed dozens of solutions to the classical construction problems of cube duplication, angle trisection, and circle quadrature. Why did all these leading mathematicians think these problems were so important that they were worth solving over and over again? I propose that Greek geometers were strongly committed to an operationalist foundational program, according to which all of mathematics including its entire ontology and epistemology is entirely based on concrete physical constructions. According to this reading, key foundational aspects of Greek geometry are directly analogous to core tenets of 20th-century operationalist, positivist, constructivist, and intuitionist philosophy of science and mathematics. Many of the motivations that drove these movements in the 20th century could very plausibly have direct parallels in ancient geometry. The operationalist interpretation enables us to view the classical problem tradition as a much more unified and coherent enterprise than has hitherto been recognised. It explains the extreme and otherwise inexplicable Greek obsession with construction problems. This interpretation goes against the abstract-logical conception of geometry favoured by ancient Platonists and modern mathematicians. But I argue that in order to understand the philosophy of mathematics of Greek mathematicians, we must be prepared to disregard philosophical sources and instead focus on the foundational assumptions implicit in ancient technical works. I analysed the complete corpus of solutions to the three classical construction problems from the point of view of such foundational purposes of constructions. I argue that technical aspects of these solutions strongly hint at such philosophical motivations, even though that is never explicit in surviving sources. This suggests new interpretations that reconstruct operationalist aspects of the solutions that were ignored or not understood by the commentators who preserved them. Notably that: Archytas's cube duplication was originally a single-motion machine; Diocles's cissoid was originally traced by a linkage device; Greek conic section theory was based on the conic compass, and in a few cases string constructions; pointwise constructions of curves were rejected.

Francesca Biagoli (Torino)

Mathematics in the relativization of the Kantian a priori: A reconsideration of Cassirers functional account.

The idea of a relativization of the Kantian a priori to the presuppositions for the possibility of scientific inquiries - first articulated by Cassirer and others in connection with the revolutions of physics in the early twentieth century - has been enjoying a revival in more recent philosophy of science. However, it is being questioned whether this view provides sufficient resources to account for scientific change. Given Cassirers commitment to the aprioricity of mathematical structures, it seems that the only strategy available to him and his followers is to establish an abstract relation of approximate inclusion between succeeding theories, and look for a different explanation of theory change in terms of communicative rationality (Friedman) or in conventional terms (Stump). This paper aims to explore a somewhat different strategy inspired by Cassirers functional model of concept formation in some key examples from his epistemological works, where the focus is on how structural procedures from nineteenth-century real analysis, analytic and projective geometry can find a fruitful expansion and a variety of applications to nonmathematical domains. Cassirer attributed a constitutive role to the functional dependencies at work in this model of concept formation, and articulated an account of objectivity that is intrinsically dynamic. The functionalization of the object of knowledge in Cassirers sense involves a specific kind of relativization, insofar as it allows one to account for the interrelations of constitutive elements at various levels (and not exclusively at the highest level of the logico- mathematical syntheses, as some of the contemporary discussions suggest). My suggestion is that Cassirers account sheds light on the role of mathematics in the relativization of the Kantian a priori, and is of particular interest in the philosophical discussion of the epistemological implications of cases where there is a cross-fertilization between different branches of mathematics, as well as a co-evolution of mathematical and physical theories.

Rami Jreige (Ecole Normale Supérieure)

How an Historical In Re account of mathematical Structuralism Sheds Light on Applicability

Mathematical structuralism is the notion that mathematics is talk of structures, not objects. The entities within the structure, e.g. numbers, are not independent of the structure they are in and cannot exist outside of it. This concept has grown in popularity over the years with many different conceptions put forth. This presentation will focus on in re structuralism, an aristotelian form of the idea that traces the emergence of mathematical structures from the concrete to the abstract.

The two main proponents of this theory, Michael Resnik and Charles Parsons, both highlight how abstract mathematical structures can emerge from the concrete, yet they both end up following W. V. Quine and relying on his indispensability argument (IA) as a link between the concrete and abstract.

What will be presented here diverges from Resnik and Parsons in terms of both their epistemology and their reliance on IA. I propose to add another layer of abstraction, the model (in the scientific sense). This addition, along with Jacob Kleins historical account of mathematical transition from the Greeks conception to the modern symbolic form it is in now, provides a possible method for the emergence of mathematical structures from the concrete. It also hints at a possible way back in terms of mathematical applicability, mirroring scientific usage.

The transition from the concrete to the model has many precedents, e.g. David Lewis way of abstraction, Parsons quasi-concrete objects, etc. However, the transition from the model to the abstract is relatively unique. This move is the most complicated as it pertains to the removal of the abstract objects from the model in general. In other words, one has to remove the objects that were being talked about in the model to talk purely of numbers, with no seeming reference. This move, as presented by Jacob Klein, can be seen to emerge historically.

Klein makes two claims: that the ancient Greeks never spoke of numbers eo ipso but rather they were always attached to the concrete via the requirement of a unit (or material or monas). Kleins second claim pertains to Vietas translation of Greek mathematics, which did not retain the units, thereby moving mathematical talk from first to second intention, understood in the medieval sense, thus paving the way for symbolic mathematics. Therefore any talk of objects in a model was dropped to reach a second intention way of talking about mathematical entities, a move that takes us from the model to the structure.

Finally, Dedekinds arithmetics axiomatisation, and Hilbert and Freges assertoric vs definitional axioms debate show that even the axioms later transitioned from including attachment to the concrete (assertoric) to purely symbolic (definitional), thereby shedding mathematics final concrete vestige.

This transition would not only show the emergence of mathematics from the concrete, but also provide a blueprint for a reverse transition from mathematical structures to applications via model, in a similar fashion to the sciences.

Karim Zahidi (Ghent University)

On the explanatory strength of proofs by mathematical induction.

Since the publication of Steiner's paper on explanation within mathematics, the topic of intra-mathematical explanation has attracted considerable attention from philosophers of mathematics and mathematical practice. In particular, the question as to what distinguishes explanatory proofs from non-explanatory proofs has been discussed extensively in the literature. While various philosophers have proposed different accounts as to what distinguishes explanatory proofs from non-explanatory ones, there has been consensus that some specific types of proofs are not explanatory. For example, there is near-unanimity that neither proofs by contradiction, nor proofs by exhaustion are explanatory. This consensus does, however, not extend to an important type of proof, to wit, proof by mathematical induction.

Much of the debate on the explanatory value of inductive proofs has focussed on a fairly restricted set of examples mainly drawn from elementary number theory. Some philosophers have ventured outside the domain of elementary number theory. Most notably Baldwin's discussion of Henkin's proof of Gödel's completeness theorem and Lehet's discussion of the use of inductive definitions, are case studies that probe the explanatory strength of induction in more advanced areas of mathematics. While these case studies enrich the discussion on the explanatory virtues of induction, they remain limited to examples in which the explanatory strength of inductive proofs is due to the fact that the entities involved are defined inductively.

In this talk I want to broaden the set of examples to include proofs of mathematical statements that do not concern inductively defined entities. In particular I will explore the explanatory value of inductive proofs in group theory by comparing an inductive proof of Cauchy's theorem with a non-inductive proof. I will argue that the inductive proof has greater explanatory strength than the non-inductive proof. This shows that the the explanatory strength of a proof does not depend on whether it uses induction or not. I will conclude with some observations as to why some inductive proofs have more explanatory value than others.

Ann Wyverkens (Ghent University)

The Intermediate Value Theorem: Proofs, Graphs, Narratives and Explanations.

Bernard Bolzano (1781-1848) proved the Intermediate Value Theorem (IVT) in a purely analytical way in 1817. In Philosophy of Mathematics. A Contemporary Introduction to the World of Proofs and Pictures (2008) James Brown claims that graphs of the IVT are proofs allowing reliable inference of the intersection point. Markus Giaquinto argues in his paper Crossing Curves: A limit to the Use of Diagrams in Proofs (2011) that on the contrary, IVT graphs are no sound proofs allowing inference of the intersection point. Both Brown and Giaquinto build their argumentations on considerations concerning continuity concepts. Different concepts of continuity play a role in the proofs and graphs. According to Brown analytical - continuity and graphical pencil continuity are deeply related. However, it is by comparing and contrasting - continuity and pencil continuity in his argument that Giaquinto shows that the existence of an IVT intersection point cannot reliably be inferred from a graph. Hence, Giaquinto concludes that IVT graphs cannot be considered genuine proofs. I agree with Giaquinto on this. Starting from the inadequacy of IVT graphs as genuine proofs, I want to make two contributions in my talk. The first is an extension of Giaquintos thesis on IVT graphs to what I call IVT visualization narratives. In his book James Brown also presents a visualization narrative of Bolzanos third theorem (which is a generalization of the IVT). Brown claims that his visualization narrative, which I will call the IVT hiking narrative is a proof of Bolzanos third theorem. Here Brown implicitly introduces a third kind of continuity, which I will call: hiking path continuity. My first aim is to show that the IVT hiking narrative cannot be considered sound proof either. So, I extend Giaquintos line of reasoning on the inadequacy of IVT graphs as proofs of the IVT theorem to the inadequacy of IVT visualization narratives as proofs of the third theorem. The position that I defend, partially based on Giaquintos arguments, is compatible with claiming that the graphs and narratives do some important psychological work as Brown notes: [E]ven if the picture merely does psychological work that in itself could only be explicable by assuming that - continuity and pencil continuity are somehow deeply related. This brings me to my second aim: I will explore the explanatory value the IVT graphs, the IVT hiking narrative and the related continuity concepts may have. I do this in order to further develop the idea of psychological work that Brown mentions. To pursue this second aim I start from the findings in Mechanistic Explanation and Explanatory Proofs in Mathematics (Frans and Weber 2014) and Is Mathematics a Domain for Philosophers of Explanation? (Weber and Frans 2017). In sum, my second aim in this talk is to show - based on what I will call Weber and Frans Woodwardian account of mathematical explanation - that the IVT graphs and the IVT hiking narrative can do more than merely illustrate the IVT. The aim is to show although they are not genuine proof they have explanatory power.

Robert Knowles (The University of Bristol)

Distinctively mathematical explanation: A deflationary proposal.

Distinctively mathematical explanations (DMEs) are explanations of physical phenomena that seem to turn on matters of pure mathematical fact. A good understanding of how DMEs work is crucial to our overall understanding of mathematics and its applicability to the natural sciences. In this paper, I present a deflationary theory of DMEs, and argue that it is best among its rivals. My theory rests on four plausible assumptions:

- Mathematical statements characterise structures that a physical system can either (approximately) satisfy or not (approximately) satisfy.
- The operative mathematical statement M in a DME states an equivalence of the form ⌜M1 if and only if M2⌝, where M1 and M2 are mathematical statements that characterise structures in their own right.
- Thus construed, M provides a distinctive kind of hypothetical knowledge about the physical world: that any physical system that satisfies (doesnt satisfy) one of M1 or M2 is one that satisfies (doesnt satisfy) the other.
- For a given DME, M will have been chosen so that any physical system satisfying one side (let it be M2) will clearly be one in which the explanandum occurs, and the fact that the target physical system satisfies the other side (let it be M1) is relatively well-understood.

My theory can be stated as follows:

The Transmission Theory (T-theory)

An explanation of phenomenon P arising in physical system S that invokes mathematical statement M is distinctively mathematical iff M improves our understanding of P by allowing us to recognise that whatever explains the fact that S satisfies M1 also explains P.

On the T-theory, the M of a DME is not itself the source of explanatory power. It does, however, play an indispensable role in improving our understanding of P, by transmitting our understanding of the fact that S satisfies M1 (a fact that is relatively well-understood) to P. The T-theory countenances a distinctive class of explanations worthy of the name distinctively mathematical, while denying that the mathematics itself contributes anything explanatory. It is in this sense that the T-theory is deflationary.

After elaborating on the T-theory and motivating it by showing that it recovers our intuitions concerning key examples, I argue that the T-theory is best among its rivals by developing the following dilemma. If we assume that the mathematical fact invoked by a given DME is itself explanatory, we face a choice: either the explanandum depends on the invoked mathematical fact by some as-yet-to-be-specified dependence relation, or the explanandum depends on non-mathematical things via some mathematical/logical relation. All extant theories of DMEs grasp one or other of these horns. I show that both horns lead to ruin: the first raises serious metaphysical and epistemological problems; the second faces the thorny challenge of explaining how a mathematical/logical relationship could be explanatory. The only way to avoid this dilemma is to go deflationary, as the T-theory does.

Jose Perez Escobar (ETH Zürich)

Paradigmatic examples of ”genuine mathematical explanations” in biology are only indicative of mathematical heuristics.

Recently, the issue of whether there are genuine mathematical explanations or distinctively mathematical explanations outside mathematics has received substantial attention. Lange argued that some explanations are distinctively mathematical because mathematical necessity, and not causes, carries most of the explanatory burden (Lange 2013). Craver and Povich have argued that those examples are better explained by causal mechanisms since there is a concrete explanatory directionality for which mathematics, being directionless, cannot account (Craver and Povich 2017). In this work, I address one of these paradigmatic examples, the honeycomb, and propose an alternative interpretation of the role of mathematics more in line with biological practices and scientific intuitions.

This interpretation entails a nominalistic reconstruction of the explanation, which does not require a mathematical object as explanans and hence does not qualify as a distinctively mathematical explanation. At the basis of this reconstruction there is a pertinent differentiation between mathematical optimization and biological optimization which, as I argue, are conflated in the mathematical explanation. The mathematical optimization argument at stake provides a rough heuristic summary of what is actually a process of biological optimization of a given evolutionary trajectory, with its actual bifurcations resulting from natural selection and the feasible biological solutions involved. However, when mathematics is taken as a constituve explanans of the biological explanandum, the intuitions at stake about mathematical optimization conflict with those featured by the standard tenets of biology.

In this context, the mathematical part of the explanation: 1) is a heuristic for a given evolutionary trajectory and depends on more central, specific teleological/functional notions in order to be explanatorily relevant, and 2) assists in the discovery of specific purposes/functions of biological phenomena. I will argue that, in biology, the role of mathematics in those paradigmatic examples of distinctively mathematical explanations is actually one of heuristic enablement of teleological/functional explanations.

I will support this alternative interpretation by extrapolating it to a similar but more illustrative case from cognitive neuroscience: the hexagonal periodicity of grid cell activity. I will show how similar mathematical insights (hexagonal shape) are used in a representative scientific practice. The case of grid cells is useful in understanding the role of the mathematical insight and of teleological/functional notions because: 1) The purposes/functions at stake are not evident as in the honeycomb example, and thus they must be addressed instead of being taken for granted and disregarded in the account of the explanation, and 2) the mathematical component was not available in 2004 (when grid cells were discovered) but only in 2005 (when the hexagonal pattern of grid cells was observed) and onwards, which offers a control situation where the epistemic role of the mathematics can be isolated and characterized.

Kristian Campbell Gonzalez Barman (Ghent University)

Distinctively Mathematical Explanations of physical facts? Some examples are dubious at best.

The debate around Distinctively Mathematical Explanations of physical facts (DMEs) is mostly framed in terms of examples. Different authors find different examples intuitively appealing, and yet these examples are the cornerstone to either admitting or rejecting the idea that there are DMEs.

I would like to argue that there is a group of very popular examples that hinders progress in the debate. I will call these theoretic examples to separate them from other groups of examples (such as optimality, asymptotic, or power-law explanations). Some of these theoretic examples include strawberry-distributions and bridge-crossings, which are among the most popular and cited examples of DMEs. These examples are sometimes used to justify accounts of DMEs. I believe this is a mistake.

My main point is that the explananda of these examples do not concern physical facts and should therefore be abandoned in so far as the debate concerns whether there can be DMEs of physical facts.

The argument starts by analyzing these examples, highlighting how a few important underlying assumptions show that their explananda are not physical. I first focus on the infamous strawberry-distribution example, considering several modifications that showcase why these assumptions (detailed below) are necessary conditions for the argument to work, to wit: Mother can distribute 1 strawberry-mobile-photograph between 3 children, and she can distribute 1 eucaryotic-dividing-cell between 2 children (material identity assumption), Mother cannot fairly distribute 24 strawberries between 3 children, since some strawberries are bigger than others (fungibility), Mother cannot distribute 3100 strawberries between 3 children, nor can she distribute 9 soap bubbles (ontic stability).

Material identity means that it is possible to identify, separate, and track each individual. Ontic stability means that the individuals relevant attributes remain within the tasks timeframe and conditions. Fungibility means that each individual is interchangeable for another.

Fungibility is a human abstraction (we can make sets of random objects). This shows that the task at hand is not physical, but mathematical: heaps are to be equinumerous. All that matters is the number of strawberries; the explanandum necessarily refers to numbers. Numbers are not physical facts. The task should be understood as a collection of abstractions pieced together to make a game. These games have intentional goals (e.g. making equinumerous heaps), initial conditions (TWENTY-THREE ontically-stable, materially-identical objects) and rules (e.g. we must not cut and should consider certain objects fungible). Given this game-configuration, a piece of mathematics is invoked to explain certain end-game configurations. But game configurations are not physical facts. The success or failure of the task is not due to physical considerations, but to whether the rules were followed (e.g. was one strawberry cloned at any point?). At best the explanation is not of a physical fact, at worst it is tautological (23 is not divisible by 3 because 23 is not divisible by 3). Similar arguments are made for other examples, e.g., there is no physical impossibility in crossing all bridges of Königsberg once (one could take a boat): this impossibility is born from the rules of the game.

Erik Weber (Ghent University)

The weird metaphysics behind distinctively mathematical explanations.

- Though many papers have been written on alleged distinctively mathematical explanations (DME) no scholar (to my knowledge) has tried to give a precise demarcation of the problem domain. I will provide an exact delineation by showing that some explananda cannot have correct causal explanations. Langes favourite example is in this class of explananda:

  Jane failed to distribute her 23 strawberries among her 3 children.

  The following explanandum is not in the domain:

  Jane failed to distribute her 24 strawberries among her 3 children.

  Acknowledging that there is an issue (viz. that some explananda do not have correct causal explanations that address them) does not imply there are legitimate DMEs. My initial characterisation of the problem is philosophically neutral: we can go in many directions, many possible solutions are possible.

- My position is that DMEs are a bad way to solve the problem. In the second part of the paper argue that, if you develop a DME to address a problematic explanandum, you have to adopt a very weird metaphysics (a mathematical ontology which I call powerful Platonism) in order to make these explanations work. If you dont accept this metaphysics, a DME does not work as an explanation. So, developing DMEs is a rather bad solution for the problem.

- In the third part of the paper I briefly indicate that there is an alternative: constructing what I call distinctively generic explanations. They do not presuppose weird metaphysics. And they use mathematics in a standard way (like it is used in causal explanation in physics).

Lieven Decock (Vrije Universiteit Amsterdam)

Freges Theorem and mathematical cognitio.

I consider the relation between two basic conceptions of the natural numbers both in formal axiom systems and in the cognitive sciences. Peano Arithmetic, based on the successor relation, is a formal expression of an ordinal conception of numbers. In Frege Arithmetic one-to-one correspondence is the central concept. Frege Arithmetics only extra-logical axiom is Humes Principle, which states that two sets are equinumerous if their elements can be put in a one-to-one correspondence relation. Freges Theorem then states that Peano Arithmetic can be interpreted in Frege Arithmetic. The epistemological implications of this result have been controversial. Starting from Hecks epistemological analysis of the conceptual relations between cardinality, counting, and equinumerosity, and expanding on his discussion of empirical results on numerical cognition, I assess the relative importance of equinumerosity and the successor relation in empirical results on numerical cognition in developmental psychology and linguistic anthropology. I discuss Gelman and Gallistels cardinality principle, Dehaenes core systems (subitizing and the approximate number system), and models of exact numbers (accumulator model and bootstrapping models), and numerical abilities in enumerate societies. I conclude that both the successor relation and one-to-one correspondence independently play a role in the development of number concepts. However, the cognitive explanations of the acquisition of numerical concepts arguably do not lead to a full understanding of numerical concepts and are in various ways problematic from a methodological point of view. In particular, in the operationalization of concept attribution subtle yet controversial decisions are made that create gaps between the operationalized concepts and the concepts in formal axioms systems. I conclude that for the analysis of numerical concepts in the mathematical practice a modest provisional apsychologism is commendable.

Michal Sochanski (Adam Mickiewicz University Poznan)

On the different ways of understanding "mathematical experiments".

In my talk I will discuss different ways in which the term experiment has been used (and can be used) to describe mathematical practices as well as the philosophical consequences of those uses.

I will first distinguish three main types of practices, in context of which the term experiment has been used: generation and analysis of instances of a general formula, testing of new ideas and experiments with representation. Several examples will be discussed, as well as subclasses of the three main classes. I will argue that those practices can be regarded as simply trial-and-error experiments, which have a common structure: first the mathematician performs some action (trying out a new method, calculation, representation type etc.), the result of which is not known in advance, which action can be seen as an experimental act; then she observes the outcome of such experiment and interprets it. I will also show how the three types of experiments differ regarding the character of the act, outcome and observation elements.

Secondly, I will discuss how the use of computers has modified the meaning of the term experiment as used in context of mathematical practice. In particular, I will analyse which of the three types of experiment discussed in the first part of talk are performed on computers and which new meanings of the notion of mathematical experiment have emerged.

Thirdly, I will distinguish two ways of arguing that the discussed mathematical practices have quasi-empirical nature. The first kind of argument goes beyond considering the discussed practices as trial-and-error procedures and draws an analogy between them and experimental practices in the natural sciences. I will analyse several such analogies by braking them up into subanalogies between aspects of the mathematical practices and elements of physical experiments. For that purpose a list of elements of laboratory experiments will be presented (partly inspired by similar list created by Ian Hacking). The second argument refers strictly to the use of computers, pointing that every application of computers can be seen as resulting in mathematical statements that are a posteriori. In this context I will also discuss the famous argument of Thomas Tymoczko that computer-assisted mathematics should be seen as experimental.

In the last part of my talk I will consider whether the experimental practices in mathematics should only be seen as belonging to the context of discovery or also to the context of justification. On the one hand paper-and-pencil experiments can be seen above all as an aspect of the former whereas experiments in the natural sciences mainly serve justificatory role. On the other hand, experiments in the natural sciences may also serve explanatory role, serving similar functions as the trial-and-error procedures of mathematics. One should also add that Tymoczko mainly discusses the experimental nature of computer proofs, while most computer practices called experiments mainly have exploratory function. I will thus conclude my talk giving some thoughts about the relation between the notions of proof and experiment in this context.

Matthew Garrett (University of Tilburg)

Deep Learning as Experimental Mathematics at Sciences Border.

SUMMARY Philosophy has given few accounts on machine learning, but those that have oer descriptions of a dynamic interaction between machine learning and philosophy of science [1], or, more strongly, machine learning as an experimental philosophy [2][3]. This paper claims that machine learning and in particular deep learning explores deep learning, oers an experimental

mathematics which similarly warrants more attention from philosophy. The experimental component is emphasised as it relates to an empirical method for mathematical discovery rather than a logical method such as automated theorem provers. This dynamic interaction between deep learning and experimental mathematics is continuing to develop, and this paper gives a conceptual and historical context for these recent trends.

PRIOR PHILOSOPHY In contrast to statistics, machine learning is rarely addressed in philosophy of science. Theoretical frameworks nonetheless exist in domain to explain methods and compare to philosophy (see e.g. [4], comparing VC theory [5] and falsificationism). Importantly, justification in machine learning can be strictly empirical from benchmark tasks [6], leaving mathematical explanation incomplete. Similarly, research is commonly motivated by informal conjectures that prioritize constructive intuition over formalism. In combination, this allows for increasing model complexity at the cost of scrutability.

Mathematical context Computers have long been a sandbox for mathematical experimentation. Following Lakatos, this can be viewed as a quasi-empirical approach to mathematics, distinct from the axiomatic approach [7]. Here, demonstration reigns over derivation. This, then, would seem more consonant with machine learning. Indeed, its once-popular designation of pattern recognition could refer as much to all of mathematics under certain philosophical accounts (e.g. [8] [9] among others). However, use of machine learning for experimentation in practice was rather limited.

Deep Learning context That is likely changing with recent advancements with deep learning. Alhough the tendencies mentioned earlierinscrutibility and informalitybecome more prominent for deep learning, so too do the possibilities of application on complex domains. These can use billions of parameters, highlighting the inscrutability. Not just for scientific discovery, it echoes a core spirit of mathematics: to find structural qualities that unify separate domains. Accordingly, recent years have seen rising engagement from mathematicians with deep learning (e.g. [10][11][12][13]).

ARGUMENT I claim that deep learning can overcome inscrutability in seeking justification in two predominant ways, both conducive to mathematical experimentation and exploration. Justification by consilience Models are not just compared on benchmark metrics, but also on tests they were not trained for. This consilience strengthens trust that the model is not merely exploit task-specific artefacts. This accords with the unifying and connective spirit of mathematical practice. **Justification by novel synthesis** The generation of novel examples (i.e. conjectures) takes the form of seeing is believing for demonstrating capacity for qualitative structure concordant with human perception. This accords with the spirit of mathematical conjecture and exploration into possibility.

That deep learning results are not fully understood makes it particularly useful for mathematicians because it allows for unexpected or unknown empirical examples. While I claim this points to wide relevance, there are important caveats, most importantly the need for large amounts of data. As such, early work is concentrated in topics such as topological structures in Big Data or theoretical physics, signifying the boundary of discovery for both mathematics and science.

Colin Rittberg (Vrije Universiteit Brussel)

On the epistemological relevance of the ethics of mathematics.

It is unsurprising that ethical issues, such as leaky pipelines or academic nepotism, arise in the social practice mathematics. In this talk I argue that some of these ethical issues are epistemologically relevant. A core insight is that having a rigorous proof of a mathematical statement is not enough. People need to be aware that you have this proof, and they need to choose to pay attention to your work. Questions about which pieces of knowledge we are exposed to and choose to engage with are entangled with issues of social power and a potential source for epistemic injustices. I present some case studies to make visible how these ethical concerns impact the epistemology of mathematics.

Nicola Bonatti (LMU München)

Extremal axioms and the reflective equilibrium of intended models.

The literature agrees that the (quasi-)categoricity of Peano arithmetic, Hilbert geometry and Zermelo-Fraenkel set theory fails to demonstrate that there is a unique structure corresponding to our informal mathematical practice see Button and Walsh (2018). In this talk, I will argue that the indented model of such theories is not determined by the categoricity theorem alone, but by the assumption of extremal axioms such as the axioms of Induction, Completeness and either Constructibility or Large Cardinals first studied by Carnap et al. (1981). The leading idea is that extremal axioms implies a reflective equilibrium between the informal beliefs concerning the subject matter of a theory and the formal resources adopted to formalize it. More precisely, the mathematician starts out with some informal beliefs about, for instance, the natural numbers which guide her trough the formalisation of arithmetic (specifically, the axiom of Induction), and then having considered the consequence of that formalisation (i.e. categoricity), use that property to further confirm her prior beliefs.

I will support my claim arguing that external axioms are both intrinsically and extrinsically justified in a specified sense close to Maddy (1997). In order to highlight the interplay between the intrinsic and extrinsic justification of extremal axioms, I will first distinguish between the pre-formal, formal and post-formal stages in the development of mathematical theories as suggested by Lakatos (1980). On the one hand, extremal axioms are intrinsically justified because they express the properties of finite chain, extendibility and (non) constructibility which are identified trough the analysis at the pre-formal stage of the basic notions of, respectively, natural number, point/line and set. On the other hand, extremal axioms are extrinsically justified because they imply at the post-formal stage the categoricity of the models (if the theory is formulated in Second-order logic), thus providing a faithful formalization of the informal beliefs. I will conclude that the reflective equilibrium between the intrinsic and extrinsic justification of extremal axioms grounds the coherence of our judgments about the intended model of such theories thus debunking the theoretical threat of non-standard models.

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Understanding (defective) mathematical theories.

If mathematical theories are continuous with empirical science, it is sensible to expect that the epistemic aims of mathematics are also continuous with those of other, less formal, scientific disciplines. Being scientific understanding one of the most valuable assets of the scientific enterprise, the achievement of it should also be an epistemic goal for mathematicians. Here I discuss whether mathematicians can achieve legitimate scientific understanding of defective theories and if so, how is this possible.

On the one hand, understanding has been traditionally considered to consist of knowledge about relations of dependence. When one understands something, one can make all kinds of correct inferences about it (Ylikoski 2013: 100). In addition, scientific understanding is often regarded to be both explanatory and factive, this is, it requires the previous achievement of explanatory knowledge and the content of understanding can only include true propositions that are known to be so. This makes it impossible to legitimately understand a knowingly defective (vague, conflicting, inconsistent, false and even impossible) set of information. On the other hand, many areas of mathematics, and formal sciences in general, concern the study of defective theories (Cf. Colyvan 2009, 2013; Friend 2014; van Bendegem 2014). Yet, despite the fact that most of these theories are knowingly defective, mathematicians have found different ways of scrutinizing and working with them to the point that they report having understood both the theories as well as the phenomena that they describe. The combination of these facts poses the following dilemma: either it is possible to achieve legitimate understanding of defective (formal) theories or the mathematicians that report having understood any defective theory are mistaken. Hence the importance of addressing both issues together.

Here I explain under which circumstances, when mathematicians report having understood a defective theory, their claim might be legitimate. I argue that mathematicians understand a theory if they can recognize the theorys underlying inference pattern(s) and if they can reconstruct and explain what is going on in specific cases of defective theories as well as consider what the theory would do if not-defective even before finding ways of fixing it.

In order to do so, I proceed in four steps. First, I characterize scientific understanding and explain its incompatibility with defective information. Second, I explain why certain epistemic practices, such as exemplification (Cf. Elgin 2017) and identifying privileged inferential patterns (Cf. Martinez-Ordaz, forthcoming), should be seen as evidence of mathematicians having achieved scientific understanding of defective theories. Third, I present a structuralist approach to scientific understanding and explain the way in which understanding of defective theories is not only possible but has been constantly achieved in mathematics and other formal disciplines. Here I also address the role that (non-classical) formal apparatuses have played in gaining such an understanding -first, by ensuring the well-behaviour of the defective information and second, by structuring the explanations that contain such an information. I illustrate the above with a case study from the Newtonian Early Calculus. Fourth, I briefly sketch a way to extend this view to explain more empirically informed cases of scientific understanding of defective theories and draw some conclusions.